exponential_multivector.wxm

A test document for Geometric Algebra with wxMaxima
contains...
Initialization
Loading of functions (intrinsic and GA specific)
Pseudoscalar definition (specifies the space dimension) and
calculation of the inverse pseudoscalar used to generate the dual of a multivector
Enumeration of the standard basis for the specified dimension

Indicate the usage of the input parameters (syntax) for function mvexp()

Initialization

(%i65)
```
reset()$
kill(all)$
stardisp:true$
stringdisp:true$
noundisp:true$
simp:true$
dotdistrib:true$
derivabbrev:true$
lispdisp:true$
```

load intrinsic (maxima or lisp) function files

(%i8)
```
load("basic")$
load("facexp")$
load("functs")$
load("scifac")$
```

batchload GA specific (maxima) function files;

(%i12)
```
ext:["wxm"]$
file_type_maxima:append(ext,file_type_maxima)$
```

(%i14)
```
batchload("gafns0")$
batchload("gafns1")$
batchload("gafns2")$
batchload("gafns3")$
batchload("gafns4")$
batchload("gafns5")$
batchload("gafns6")$
```

batchload GC specific (maxima) function files;

(%i21)
```
batchload("gcfns1")$
batchload("gcfns2")$
batchload("gcfns3")$
```

the pseudoscalar and its inverse
the lowest useable dimension pseudoscalar should be {e1,e2} i.e. Plen = 2

(%i24)
```
Pseudos:{e1,e2,e3}$
Pvar:listofvars(Pseudos)$
Plen:length(Pvar)$
I:Pseudos$
ni:(Plen-1)*Plen/2$
Ii:(-1)^ni*I$
kill(ni)$
ldisplay(Pvar)$
```

(%t31) Pvar=[e1,e2,e3]

initialize this list with the only list we have so far; Pvar

(%i32)
```
lstbases:makelist(Pvar[n],n,1,Plen)$
```

the integer array, nbases (n:0,...3) is used for grader(M) in gafns4.wxm;
in order to use the same indices for the lists, (n:1,...3) define it using function array()

(%i33)
```
array(nbases,Plen)$
eset:setify(Pvar)$
for ng:1 thru Plen do
block(nbases[ng]:combination(Plen,ng),
lstbases[ng]:full_listify(powerset(eset,ng)))$
maxnbases:(combination(Plen,floor(Plen/2)))$
nbases[0]:1$ /*an array index zero cannot be used to index any of the lists*/
ldisplay(lstbases)$
kill(eset,ng)$
```

(%t38) lstbases=[[[e1],[e2],[e3]],[[e1,e2],[e1,e3],[e2,e3]],[[e1,e2,e3]]]

initialize this list again with Pvar

(%i40)
```
lstblds:makelist(Pvar[n],n,1,Plen)$
```

the list named lstblds is used for grader(M) in gafns4.wxm;
lstblds is an list of lists of blades and allblds is a list of all blades

(%i41)
```
for ng:1 thru Plen do
block(lstb:lstbases[ng],
lstblds[ng]:makelist(list2blade(lst),lst,lstb))$
allblds:[]$
for ng:1 thru Plen do
block(allblds:append(allblds,lstblds[ng]))$
ldisplay(lstblds)$
ldisplay(allblds)$
kill(lstb,ng)$
```

(%t44) lstblds=[[{e1},{e2},{e3}],[{e1,e2},{e1,e3},{e2,e3}],[{e1,e2,e3}]]
(%t45) allblds=[{e1},{e2},{e3},{e1,e2},{e1,e3},{e2,e3},{e1,e2,e3}]

end of Initialization

exponential function...
the first input is any multivector M (including scalar, a)
the second input is a non-negative integer, the highest index
in the exponential sum (e.g. n=3 for full grade algebraic multivectors or
n=7 for numerical bivectors gives an accuracy of 4 decimal places)

show that the input index is tested and that scalar, a, is handled

(%i47)
```
mvexp(a,0);
mvexp(a,1);
mvexp(a,-1);
mvexp(a,1.1);
mvexp(a,5);
```

(%o47) 1
(%o48) $a+1$
(%o49) false
(%o50) false
(%o51) $\frac{a^5+5*a^4+20*a^3+60*a^2+120*a+120}{120}$

tests with simple albebraic bivectors

(%i52)
```
B:b*{e1,e2}$
mvexp(B,2);
```

(%o53) $\frac{2*b*\{e1,e2\}-b^2+2}{2}$

(%i54)
```
C:c1*{e1,e2}+c2*{e2,e3}$
mvexp(C,3);
```

(%o55) $-\frac{c2*(c2^2+c1^2-6)*\{e2,e3\}+c1*(c2^2+c1^2-6)*\{e1,e2\}+3*(c2^2+c1^2-2)}{6}$

the tests with mixed grade multivectors are also a strong test for the multivector
geometric product, &*, since it is used repeatedly in the exponential code

(%i56)
```
lstA:[2,3]$
nameA:"A"$
makelistgrademv(nameA,lstA)$
ldisplay(A)$
```

(%t59) $A=a_{2,3}*\{e2,e3\}+a_{2,2}*\{e1,e3\}+a_{3,1}*\{e1,e2,e3\}+a_{2,1}*\{e1,e2\}$

(%i60)
```
mvexp(A,0);
mvexp(A,1);
```

(%o60) 1
(%o61) $a_{2,3}*\{e2,e3\}+a_{2,2}*\{e1,e3\}+a_{3,1}*\{e1,e2,e3\}+a_{2,1}*\{e1,e2\}+1$

(%i62)
```
mvexp(A,2);
```

(%o62) $(-2*a_{2,1}*a_{3,1}*\{e3\}+2*a_{2,3}*\{e2,e3\}+2*a_{2,2}*a_{3,1}*\{e2\}+2*a_{2,2}*\{e1,e3\}+2*a_{3,1}*\{e1,e2,e3\}+2*a_{2,1}*\{e1,e2\}-2*a_{2,3}*a_{3,1}*\{e1\}-a_{3,1}^2-a_{2,3}^2-a_{2,2}^2-a_{2,1}^2+2)/2$

(%i63)
```
mvexp(A,3);
```

(%o63) $-(6*a_{2,1}*a_{3,1}*\{e3\}+a_{2,3}*\left(3*a_{3,1}^2+a_{2,3}^2+a_{2,2}^2+a_{2,1}^2-6\right)*\{e2,e3\}-6*a_{2,2}*a_{3,1}*\{e2\}+a_{2,2}*\left(3*a_{3,1}^2+a_{2,3}^2+a_{2,2}^2+a_{2,1}^2-6\right)*\{e1,e3\}+a_{3,1}*\left(a_{3,1}^2+3*a_{2,3}^2+3*a_{2,2}^2+3*a_{2,1}^2-6\right)*\{e1,e2,e3\}+a_{2,1}*\left(3*a_{3,1}^2+a_{2,3}^2+a_{2,2}^2+a_{2,1}^2-6\right)*\{e1,e2\}+6*a_{2,3}*a_{3,1}*\{e1\}+3*\left(a_{3,1}^2+a_{2,3}^2+a_{2,2}^2+a_{2,1}^2-2\right))/6$

numerical tests for several well known bivector angles

(%i64)
```
bv:{e1,e2}*%pi/3$
mvexp(bv,7);
```

(%o65) $-\frac{\pi*\left(\pi^6-378*\pi^4+68040*\pi^2-3674160\right)*\{e1,e2\}+21*\left(\pi^6-270*\pi^4+29160*\pi^2-524880\right)}{11022480}$

(%i66)
```
expand(ev(%,numer));
```

(%o66) $0.86602127165637*\{e1,e2\}+0.49996456532891$

(%i67)
```
theta:%pi/4$
ibv:{e2,e3}$
mvexp(ibv*theta,7);
```

(%o69) $-\frac{\pi*\left(\pi^6-672*\pi^4+215040*\pi^2-20643840\right)*\{e2,e3\}+28*\left(\pi^6-480*\pi^4+92160*\pi^2-2949120\right)}{82575360}$

(%i70)
```
expand(ev(%,numer));
```

(%o70) $0.70710646957518*\{e2,e3\}+0.70710321482285$

(%i71)
```
theta:%pi/2$
ibv:{e1,e3}$
mvexp(ibv*theta,9);
```

(%o73) $(\pi*\left(\pi^8-288*\pi^6+48384*\pi^4-3870720*\pi^2+92897280\right)*\{e1,e3\}+18*\left(\pi^8-224*\pi^6+26880*\pi^4-1290240*\pi^2+10321920\right))/185794560$

(%i74)
```
expand(ev(%,numer));
```

(%o74) $1.000003542584286*\{e1,e3\}+2.4737276364728002\,10^{-5}$