VAGC_polar_grad.wxm

A development document for Geometric Algebra using wxMaxima

Exercise 5.20, VAGC page 70 for the gradient in polar coordinates for 2D only

Initialization

```
(%i40) ext:["wxm"]$
       file_type_maxima:append(ext,file_type_maxima)$
       batchload("initialize_fns")$
```

the pseudoscalar and its inverse
the lowest useable dimension pseudoscalar should be {e1,e2} i.e. Plen = 2
e.g. for four dimensions edit Pseudos:{e1,e2,e3}$ to Pseudos:{e1,e2,e3,e4}$

```
(%i1)  Pseudos:{e1,e2}$
       Pvar:listofvars(Pseudos)$
       Plen:length(Pvar)$
       I:Pseudos$
       ni:(Plen-1)*Plen/2$
       Ii:(-1)^ni*I$
       kill(ni)$
       ldisplay(Pvar)$

  (%t8) Pvar=[e1,e2]

(%i9)  batchload("initialize_lsts")$

  (%t9) lstblds=[[{e1},{e2}],[{e1,e2}]]
(%t10) allblds=[{e1},{e2},{e1,e2}]
(%t11) invblds=[{e1},{e2},-{e1,e2}]
```

end of Initialization

set derivabbrev:false$

```
(%i12) derivabbrev:false$
```

Exercise 5.20
VAGC page 70

form the coordinate vector, x from the lists of coefficients in 2D only!

```
(%i13) xstr:"x"$
       xlst:lstvector(xstr)$
       ldisplay(xlst)$

  (%t15) xlst=[x1,x2,0]

(%i16) xv:makevector(xlst)$
       ldisplay(xv)$

  (%t17) xv={e2}*x2+{e1}*x1

(%i18) x1:r*cos(th)$
       x2:r*sin(th)$
       x:ev(xv)$
       ldisplay(x)$

  (%t21) x={e2}*r*sin(th)+{e1}*r*cos(th)
```

only wr is a unit vector of wjbasis

```
(%i22) wr:diff(x,r)$
       wth:diff(x,th)$
       ldisplay(wr,wth)$

  (%t24) wr={e2}*sin(th)+{e1}*cos(th)
  (%t25) wth={e2}*r*cos(th)-{e1}*r*sin(th)

(%i26) wjbasis:[wr,wth]$
```

find the reciprocal of the basis with the usage of reciprocb() given in gafns5.wxm

```
(%i27) bld:list2vecouter(wjbasis)$
       denext:extract(bld)$
       normden:trigsimp(denext[2])$
       wkbasis_numerators:reciprocb(wjbasis)$
       wkr:wkbasis_numerators[1]/normden$
       wkth:wkbasis_numerators[2]/normden$
       ldisplay(wkr,wkth)$

(%t33)/R/ wkr={e2}*sin(th)+{e1}*cos(th)
                                  {e1}*sin(th)-{e2}*cos(th)
(%t34)/R/ wkth=-------------------------------------------
                                          r

(%i35) wkbasis:[wkr,wkth]$
```

Exercise 5.20, part a) use Equation (5.18)

```
(%i36) gradF:'wkr*'diff(F,r)+'wkth*'diff(F,th)$
       ldisplay(gradF)$

  (%t37) gradF=wkth*('d/d*th*F)+wkr*('d/d*r*F)
```

identify the unit base vectors

```
(%i38) rhat:wr$
       thetahat:ratsimp(wth/r)$
       ldisplay(rhat,thetahat)$

  (%t40) rhat={e2}*sin(th)+{e1}*cos(th)
  (%t41) thetahat={e2}*cos(th)-{e1}*sin(th)
```

inspection of the equation for gradF

```
(%i42) is(equal(wkr,rhat));
       is(equal(wkth,thetahat/r));

  (%o42) true
  (%o43) true
```

Exercise 5.20, part b)

find the lhs of part b) as grad(F) for scalar F = f(r,th)

```
(%i44) F:f(r,th)$
       ev(gradF,diff)$
       lhs:facsum(%,allblds);

(%o46) -({e1}*('d/d*th*f(r,th))*sin(th)-r*('d/d*r*f(r,th))*cos(th))-{e2}*
(r*('d/d*r*f(r,th))*sin(th)+('d/d*th*f(r,th))*cos(th))/r
```

find the rhs of part b) with rhat and thetahat

```
(%i47) diff(f(r,th),r)*rhat+(1/r)*diff(f(r,th),th)*thetahat$
       rhs:facsum(%,allblds);

(%o48) -({e1}*('d/d*th*f(r,th))*sin(th)-r*('d/d*r*f(r,th))*cos(th))-{e2}*
(r*('d/d*r*f(r,th))*sin(th)+('d/d*th*f(r,th))*cos(th))/r

(%i49) is(equal(lhs,rhs));

(%o49) true
```

&. and &^ will need special treatment when F is a vector field

```
(%i50) F:f(r,th)*rhat+fth(r,th)*thetahat$
       dFr:diff(F,r);
       dFth:diff(F,th);

(%o51) ('d/d*r*fr(r,th))*({e2}*sin(th)+{e1}*cos(th))+('d/d*r*fth(r,th))*
({e2}*cos(th)-{e1}*sin(th))
(%o52) ('d/d*th*fr(r,th))*({e2}*sin(th)+{e1}*cos(th))+fth(r,th)*
(-{e2}*sin(th)-{e1}*cos(th))+('d/d*th*fth(r,th))*({e2}*cos(th)-{e1}*sin(th))+fr(r,th)
*({e2}*cos(th)-{e1}*sin(th))
```

Exercise 5.20, part c)

```
(%i53) wkr&.dFr+wkth&.dFth$
       expand(%)$
       divF:trigsimp(%)$
       ldisplay(divF)$

                                  'd/d*th*fth(r,th)+r*('d/d*r*fr(r,th))+fr(r,th)
(%t56) divF=-----------------------------------------------------------------
                                                r
```

differentiation of a product and inspection of the rhs of the equation for divF

```
(%i57) diff(r*fr(r,th)),r);

(%o57) r*('d/d*r*fr(r,th))+fr(r,th)
```

Exercise 5.20, part d)

```
(%i58) wkr&^dFr+wkth&^dFth$
       expand(%)$
       trigsimp(%)$
       curlF:facsum(%,allblds)$
       ldisplay(curlF)$

                                  {e1,e2}*(r*('d/d*r*fth(r,th))-'d/d*th*fr(r,th)+fth(r,th))
(%t62) curlF=-----------------------------------------------------------
                                                r
```

show that rhat^thetahat = e1~e2

```
(%i63) wr&^(wth/r);
       rhat&^thetahat;
       trigsimp(%);

(%o63)/R/ {e1,e2}*sin(th)^2+{e1,e2}*cos(th)^2
(%o64)/R/ {e1,e2}*sin(th)^2+{e1,e2}*cos(th)^2
(%o65) {e1,e2}
```

differentiation of a product and inspection of the rhs of the equation for curlF

```
(%i66) diff((r*fth(r,th)),r);

(%o66) r*('d/d*r*fth(r,th))+fth(r,th)
```